AIAA 80-1847R

Fuel Efficiency of Small Aircraft

B. H. Carson*
U. S. Naval Academy, Annapolis, Md.

There is a basic mismatch between the amount of power installed in small propeller-driven aircraft and that required for efficient cruising, which results from climb performance requirements. It is shown in this paper that there is a way of using excess power for most efficient cruise, the resulting airspeed coming closest to the Gabrielli-von Kármán limit line of vehicular performance. A survey of 111 light aircraft was conducted, and it is found that many are operated at this optimum, while many more are not. A figure of merit is developed that measures cruise performance. Rationale is presented that is directly applicable to design for cruise efficiency.

Introduction

IT is safe to say that western life styles will continue to undergo change as a result of ever-increasing pressure to conserve or better utilize fossil fuels. This trend began with the oil embargo of 1973-74, when, for the first time since World War II, fuel supplies in most western nations fell significantly below established demands. In this country, speed limits have been imposed on highway vehicles, and automobile manufacturers have been compelled, both by the buying public and government regulation, to produce more fuel-efficient cars. As this situation develops, stronger measures, including heavy fuel taxation and even fuel rationing, may be anticipated.

Thus far, however, no such restrictions have been placed on the general aviation community, nor have aircraft manufacturers shown much initiative in bringing forth the aeronautical equivalent of the compact car. This is due at least in part to the fact that the entire general aviation fleet consumes a miniscule fraction of petroleum distillates as compared to that used for automotive purposes. But this situation may not persist; private flying is still largely perceived by the general public as a highly visible recreational activity, roughly in the same category as pleasure boating, and accordingly, consumptive of valuable resources which could well be put to better use. Few people realize the extent to which general aviation contributes to our national transportation system. Figures released for 1978[1] show that the U.S. general aviation fleet flew 4.5 billion miles for this period, roughly twice the number flown by all domestic airlines, and that general aviation aircraft transported 29% of all people who traveled intercity by air in the United States.

It was with these thoughts that the present study was undertaken. If general aviation is to continue without restriction, there must be a concerted attempt to design future aircraft with fuel efficiency as an uppermost consideration. It is believed that the results of this study will provide new insights as to the direction in which these design efforts should proceed.

To keep things within reasonable bounds, it was decided to center this study around a particular group of general aviation aircraft, which are those limited to one or two reciprocating engines and gross weights less than about 8000 lb (35 kN). This is not overly restrictive; this grouping (which will henceforth be referred to as "private aircraft") includes 92% of the 200,000 general aviation aircraft presently in service in the United States.

Presented as Paper 80-1847 at the AIAA Aircraft Systems and Technology Meeting, Anaheim, Calif., Aug. 4-6, 1980; submitted Sept. 11, 1980; revision received Sept. 17, 1981.

*Professor, Aerospace Engineering. Member AIAA.

Vehicle Efficiency

As an overview, we begin with a short discussion on the subject of vehicle efficiency.

The definition of vehicle efficiency, as means of determining which vehicle is "best," is a subject that has intrigued engineers for many years, and various measures of this have been devised. The most often cited rating is the transport efficiency WV/P, where W is the vehicle weight, V the speed, and P the installed power of the vehicle.

Jewell[2] has made studies of many vehicles and concludes that "some form of transport efficiency is a significant measure of vehicle worth." However, objections can be raised that an evaluation of a vehicle's worth based on this rating is misleading. First, it turns out that all vehicles belonging to a generic class have about the same transport efficiency. Thus transport efficiency is most appropriately used when comparing classes of vehicles, rather than specific designs within a given class. Second, using transport efficiency as a primary figure of merit completely obscures the fact that some vehicles are manifestly suited for some transportation needs, and as equally unsuited for others; witness the jet transport and the supertanker.

In the wide spectrum of transportation modes, aircraft are unique in that they are capable of developing high speeds economically. This permits human participation in distant events with a minimum investment in travel time. The actual worth of speed is difficult to define in any generally acceptable sense; those involved in commerce measure it in one way, and vacationers in an altogether different but equally valid way. About the best that can be said is that speed is worth "something," which is in turn related to the worth of time. And aircraft are the only vehicles that can offer high speeds, at any price.

This is an important fact to keep in mind when aircraft are compared with other vehicles, such as automobiles, from the standpoint of fuel efficiency, as was recently done.[1] In this regard it appears that private aircraft are generally competitive with medium sized passenger cars. While such a comparison is useful in that it can be readily comprehended by the lay public, it suffers from the fact that the worth of speed has not been taken into account; even a mediocre private aircraft flies twice as fast as a passenger car cruises.

What is needed is a definition of aircraft efficiency unique to aircraft, and based upon the principles of flight. Only in this way can the merits of a given design be determined or may we gain an appreciation for the factors that govern aircraft fuel efficiency.

The Lift-Drag Ratio

In seeking a definition of aircraft efficiency, it is natural to begin with the lift-drag (L/D) ratio. Traditionally, its maximum value has been accepted as a primary figure of merit for a given aircraft. When expressed in reciprocal form, it is akin to the ordinary friction coefficient encountered in mechanics.

The L/D ratio enters aircraft performance calculations in an obvious way. Neglecting speed variations in propeller efficiency (a reasonable assumption for aircraft having constant speed propellers) it can be shown that the speed which maximizes range, neglecting wind, is that corresponding to the best L/D for a propeller-driven aircraft. This is established by noting that, since the time rate of energy expenditure dE/dt is proportional to power, which is in turn proportional to the product of drag and velocity, i.e.,

$$dE/dT \sim P \sim DV$$

then $dE/ds=(dE/dt)(dt/ds)=P/V=D=W(L/D)$, where s is the distance flown. Therefore the rate of energy expenditure per unit distance is minimum when L/D is maximum, indicating that in a comparison between two aircraft, the one having the higher L/D (all other things equal) ought to be judged the more efficient.

It is a practical fact, however, that aircraft are almost never flown at the airspeed corresponding to the maximum L/D ratio. Pilots operate aircraft according to manufacturers' instructions, where the power setting (i.e., percent of rated power) is treated as the independent variable. The airspeeds corresponding to these settings (typically, 65% and 75%) are usually well in excess of the theoretical optimum.

An objective observation is that aircraft are designed with a basic mismatch between the aerodynamics of the airframe and the amount of power required to realize its most efficient use, and that as a result, aircraft are operated in a wasteful fashion. Before making any hasty judgments regarding pilots and designers, however, we should explore the reasons why this practice exists. For the present, let it simply be noted that the L/D ratio does not appear to be as useful a measure of aircraft efficiency as intuition might suggest.

Practical Considerations

The mismatch between the amount of power installed in aircraft and that required for efficient cruise derives from a variety of sources, chief of which is the fact that aircraft must be able to climb. This requires a considerable amount of excess power, which when integrated in time, results in potential energy and hence the altitude required for efficient cruising operations. Once at altitude, this excess power then becomes available for high speed cruising. That this power is used for such purposes reflects not only man's desire for speed, but also the fact that many of the costs associated with flight are prorated on flight time. Thus the cost of the additional fuel required to cruise at off-optimum airspeed is usually more than compensated for by the savings in these other costs. There is also the belief, probably rooted in fact, that "throttling back" to the optimum airspeed during normal operations is a false economy, since the engine will not run at its best operating temperature, and wear will be accelerated.

This has evidently been true since the beginnings of powered flight. Hoerner[3] has made an analysis of the 1903, 12-hp Wright biplane, and concluded that this aircraft flew at or near its optimum lift coefficient for gliding, i.e., at maximum L/D. "Other flights," he notes, "were made in airplanes of much the same dimensions, with the aid of 'stronger' (27 hp) engines." Even the Wrights were evidently dissatisfied with an aircraft engine sized to optimum airspeed, and remedied the situation with more power! Thus, among their other notable achievements, they discovered a design truism still valid today: If an aircraft engine is sized to optimum airspeed, pilots will decide that the aircraft is underpowered.

Design for High L/D at Cruise

Having noted that private aircraft are equipped with engines far outsized for cruising at optimum airspeeds, the question that next comes to mind is this: Why are aircraft not designed aerodynamically to produce high L/D at acceptably high airspeeds?

To answer this, we begin with what might be called the "designer's dilemma," the basic problem of sizing an aircraft to meet a cruise specification. Without being overly restrictive, we confine our attention to aircraft whose drag/lift ratio may be expressed by the simple relation (polar) having the form

$$D/L = AV^2 + B/V^2 \tag{1}$$

where V is the flight speed, and A and B are lumped parameters† defined as

$$A = \rho f/2W \tag{2}$$

$$B = 2W/\rho b^2 \pi e \tag{3}$$

Here, ρ is the air density, f the equivalent parasite area for the aircraft (a fictitious area which, when multiplied by the freestream dynamic pressure, equals the viscous drag), W is the weight, b the span, and e the airplane, or "Oswald's" efficiency factor which accounts for departures from theory in the calculation of induced drag.

To achieve low D/L at some specified airspeed, both A and B should be held as small as possible. But close inspection shows that this is not easily done; any attempt at reducing A by reducing ρ or increasing W results in a corresponding increase in B, and vice versa. The same tends to be true for the two fictitious areas f and b^2, since they exist in some direct proportionality, and the value of e tends to be rather inflexible, ranging from about 0.6 to 0.8 for most designs. Therein lies the dilemma.

The design decision as to the magnitudes of A and B can be postponed temporarily by noting that, for a given A, B, the above expression yields a minimum in velocity. Calculating this, it turns out that

$$\hat{V} = (B/A)^{1/4} = (2W/\rho)^{1/2}(\pi b^2 fe)^{-1/4} \tag{4}$$

$$(\widehat{D/L}) = 2\sqrt{AB} = 2(f/\pi b^2 e)^{1/2} \tag{5}$$

where the circumflex refers to minimum D/L, or maximum L/D.

This only serves to bring the dilemma into sharper focus. The requirement for high $\hat{V}$ dictates a low product of f and b^2, whereas high L/D requires a low *ratio* of f and b^2. Thus L/D and airspeed are inimical. If high L/D is the sole criterion, it can indeed be achieved: L/D ratios of thirty or more are not uncommon for modern sailplanes. The drawback is that the corresponding airspeeds are unacceptably low for cruising aircraft. Contemporary sailplanes achieve their high L/D's at airspeeds approximating 50 knots.

The real restriction on high L/D at cruise can be deduced by treating the maximum L/D ratio and the corresponding airspeed as independent variables, treating ρ, W, and e as constants, and solving for f and b^2. This yields

$$f = W/(\rho\Lambda\hat{V}^2) \qquad b^2 = 4(W\Lambda/\pi\rho e)/\hat{V}^2$$

†In this paper, we depart from the convention of representing aircraft parameters in coefficient form. For present purposes, the representation of performance in the actual physical variables of the aircraft serves to identify certain dependencies which would be obscured in the customary (coefficient) representation.

where $\Lambda = (\hat{L/D})$. Now, specifying $\hat{V}$ and Λ fixes b, and ostensibly, f as well. However, f is a state-of-the-art parameter, and is not amenable to simple specification. Present indications are that a value of f less than about 3 ft^2 (0.28 m^2) cannot be achieved for any production aircraft, and this only with the greatest attention to aerodynamic design. A very "dirty" design, characterizing most simple training aircraft, will normally have an f approximately twice this value.

With this constraint, it is not surprising to find on the basis of a survey (discussed later) that light aircraft designers appear to have standardized on maximum L/D's of about 10 to 14, with only a few designs lying outside these rather narrow limits. However, as previously noted, few aircraft are flown even at these modest values, cruise L/D's of 10 being typical.

Most Efficient Use of Excess Fuel

Thus far, in an attempt to develop a rational basis for determining the fuel efficiency of small aircraft, we have identified several practical aspects, summarized as follows: 1) Aircraft fuel economy is directly proportional to L/D ratio, but high L/D ratios are realized only at unacceptably low airspeeds, within present technology constraints. 2) Because of the installed power required for good climb performance, aircraft are normally operated at airspeeds well in excess of optimum. 3) Since aircraft normally operate at off-optimum airspeeds, there is obviously a fuel penalty involved.

Recalling that fuel efficiency is central to this study, it is important to determine what the penalty is for off-design cruising. Accepting that excess fuel is to be traded off for airspeed during normal operations, we will now show that there is a method of operation which represents the "least wasteful way of wasting" fuel.

By neglecting minor variations in propeller efficiency and specific fuel consumption that may exist at different airspeeds and power settings for any given aircraft, it is not difficult to calculate the theoretical amount of additional fuel that would be consumed on a given stage length if an aircraft were flown at an airspeed different from the optimum. Only the no-wind condition will be considered; the results can be extended later to include wind, as the need arises. To do this, we begin with Eqs. (1), (4), and (5), i.e.,

$$(\hat{D/W}) = A\hat{V}^2 + B/\hat{V}^2 \quad \text{and} \quad \hat{V} = (B/A)^{1/4}$$

This condition is taken as a reference, since as previously noted, it represents the optimum (most fuel efficient) flight condition.

The unit power $P_s = P/W$ required at this airspeed is

$$\hat{P}_s = A\hat{V}^3 + B/\hat{V} = 2(AB^3)^{1/4}$$

and thus, at any other airspeed V, it results after some algebra that

$$P_s/\hat{P}_s = \tfrac{1}{2}[(V/\hat{V})^3 + \hat{V}/V]$$

Now suppose that $V = \alpha\hat{V}$, where α is a number of unit order. Then

$$P_s/\hat{P}_s = \tfrac{1}{2}[\alpha^3 + 1/\alpha] = F/\hat{F}$$

where F is the fuel flow rate, assumed to scale directly with power according to previous assumptions. For a given cruise distance, the ratio of no-wind flight times will be $t/\hat{t} = \hat{V}/V = 1/\alpha$, and hence the ratio of total fuel expended for either case will be

$$(F/\hat{F})(t/\hat{t}) = \tfrac{1}{2}[\alpha^2 + 1/\alpha^2] \equiv \omega$$

Calling α an "excess speed factor," and ω an "excess fuel factor," then the ratio ω/α is a measure of the price (measured in fuel) per unit of airspeed greater or less than the optimum for a given distance flown. This quantity has a minimum, i.e., $d(\omega/\alpha)/d\alpha = 0$, when $\alpha \equiv \alpha^* = 3^{1/4}$.

It thus turns out that the best rate of return on excess fuel expended, as measured in additional airspeed will occur when

$$V^* = 3^{1/4}\hat{V} \sim 1.32V, \qquad P_s^* = (2/3^{1/4})\hat{P}_s \sim 1.52\hat{P}_s$$

$$\omega = 2/3^{1/2} \sim 1.16, \qquad t^* = (1/3^{1/4})\hat{t} \sim 0.76\hat{t}$$

indicating that a 32% increase in airspeed above the optimum will result in only a 16% increase in total fuel used. This requires a 52% increase in power. In return for this, the flight time will be reduced by 24%. This is clearly the best return in airspeed increase (and hence reduction in flying time) on excess fuel, and as such, must be regarded as the "least wasteful way of wasting," i.e., the most productive use of excess fuel for cruising purposes.

This result is remarkable on two counts: First, it indicates that the common practice of operating piston aircraft at airspeeds about 30% higher than the optimum has a rational basis, although the practice itself has probably evolved empirically. Second, the condition $V = 3^{1/4}\hat{V}$ is known to correspond to the optimum (no-wind) airspeed for *jet propelled* aircraft, in which fuel energy is converted directly into thrust (in contrast to propeller driven aircraft, in which energy is converted first to power, then to thrust) and corresponds to the minimum value of D/WV for the aircraft.

We can determine the physical significance of this airspeed for propeller aircraft from the following considerations. Assuming steady state, we can write

$$D/WV = Dt/Ws = Pt/WsV \sim E/WsV$$

which shows that the airspeed corresponding to minimum D/WV minimizes the total energy E required to transport a weight W through a distance s at velocity V. Alternately,

$$D/WV = DV/WV^2 \sim P/WV^2$$

i.e., the airspeed corresponding to minimum D/WV minimizes the power required to maintain the kinetic energy of the aircraft in the face of continuous energy dissipation by drag forces. It may be noted that since E (the total energy consumed in a given distance at cruise) is proportional to cost, then E/V is the cost of travel over a given distance per unit of airspeed, and the analysis shows that this cost will be minimized when D/WV is minimized. Thus $\hat{V}$ minimizes cost per unit distance, while V^* minimizes cost per unit of speed.

Finally, we note in passing that the reciprocal of D/WV is just the product of the airspeed and the L/D ratio. Obviously, minimizing D/WV results in maximizing this product. This is evidently the best compromise between high L/D and high speed, shown previously to exist in reciprocal relation. It is in this fashion that designers appear to reconcile the design dilemma.

The Gabrielli-von Kármán Limit

Earlier, it was noted that aircraft are unique vehicles, in that only aircraft are capable of achieving high speeds economically. In our quest for a realistic measure of aircraft efficiency, we have examined a number of factors, some practical and some theoretical, and have been led to the conclusion that aircraft are indeed designed and operated according to this philosophy, i.e., in a way that maximizes speed per unit cost, rather than distance per unit cost.

This brings to mind a celebrated paper published in 1950 by Gabrielli and von Kármán,[4] who undertook a study of many vehicles in an attempt to establish exactly this type of

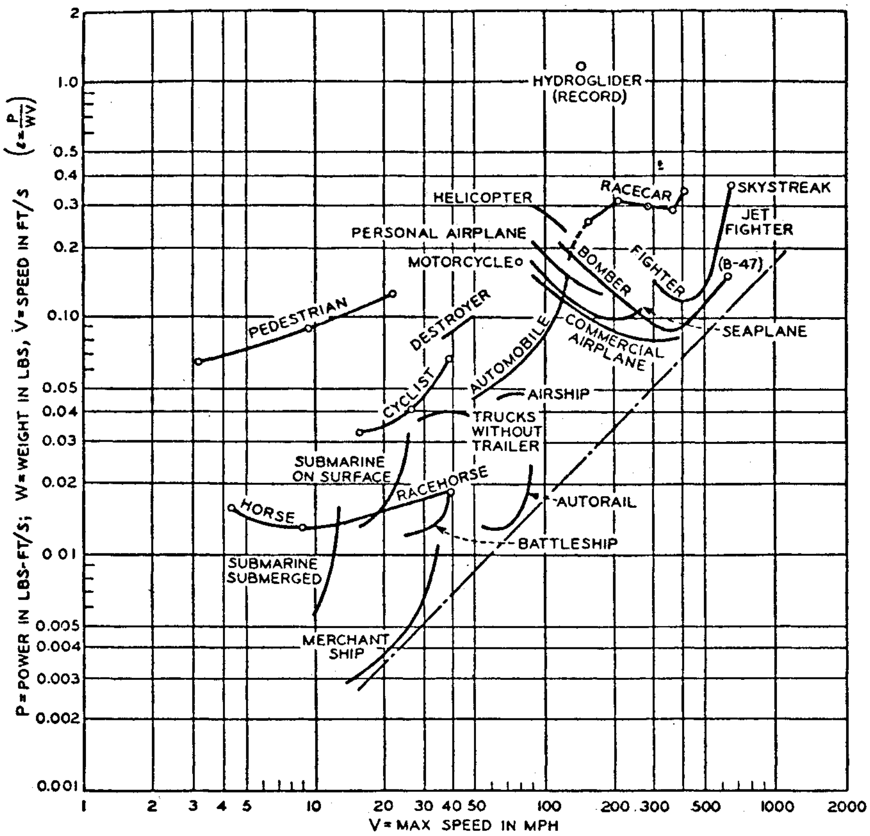

Fig. 1 Specific resistance of single vehicles[4] (used by permission of the American Society of Mechanical Engineers).

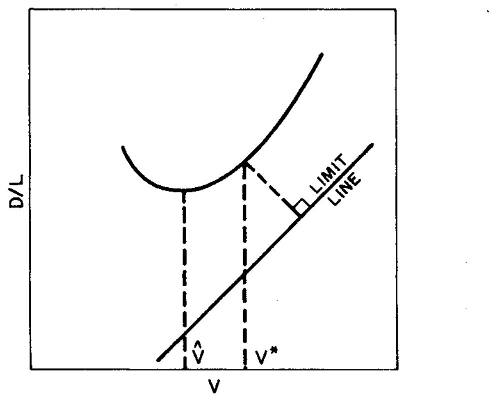

Fig. 2 Typical aircraft D/L characteristic vs V. Note velocity of closest approach to limit line is not $(V)_{L/D\,\max}$.

relationship. The paper was titled, aptly, "What Price Speed?"

Their work showed that there is an apparent technology barrier (which they cautiously referred to as "almost a kind of universal law") that sets a limit on the speed of any vehicle, regardless of type or operating medium, for a given power-to-weight ratio. By plotting vehicle specific power ϵ (defined as P/WV and hence the reciprocal of transport efficiency) against speed for numerous vehicles, it was found that there is a line of demarcation in vehicular performance between that which is feasible, and that which appears to be impossible. This is shown in Fig. 1. This line has since been known as the "Gabrielli-von Kármán limit," or, simply the "limit line," and is a standard by which all vehicles are compared; the closer the vehicle's performance lies to this line, the greater efficiency will be attributed to the vehicle. It should be noted that this limit was not derived, but rather was postulated according to the evidence developed in their exhaustive study. The authors themselves speculated on its origin, indicating that it might be related to material properties.

Returning to the matter at hand, it can be seen that the vehicle specific power for an aircraft is just its D/L ratio. Accordingly, it might be supposed that the best aircraft efficiency in the Gabrielli-von Kármán (GvK) sense would occur at a speed corresponding to best L/D. However, if the entire D/L characteristic of the aircraft is plotted in the ϵ-V coordinate system, it can be seen, as shown in Fig. 2, that the speed lying closest to the limit line is not that for minimum ϵ (best L/D), but rather a speed somewhat higher than this. It is easily shown (the details are straightforward) that this "velocity of closest approach" is identical to the speed designated as V^* in the previous development. Note that this is true regardless of whether the aircraft is powered by a jet engine or a propeller. Thus design practice, operational practice, and the limit line have been unified into a coherent whole. We have shown that it is neither practical to design for high L/D, nor to operate aircraft at the airspeed corresponding to maximum L/D. Then it was shown that if aircraft are not to be operated at the most fuel efficient airspeed, then there is a "next best" airspeed corresponding to minimum outlay in extra fuel per increment of additional speed (or decrease in flight time); and this in turn is identical to the airspeed lying closest to the GvK limit line.

This airspeed may be regarded as the cruise-optimum, and will be referred to as such subsequently.

Definition of Cruise Efficiency

It is now possible to develop a figure of merit for an aircraft, based on the closeness of its performance to the GvK limit. Before this is undertaken, however, it will prove convenient to transform the GvK coordinates into another system.

The limit line appears as a linear function of velocity in the ϵ-V plane. Thus $\epsilon/V=K$ (a constant) along the limit line,‡ so that if ϵ/V is plotted against V, there results a line parallel with the V axis. This corresponds to an invariant limit, and all points to the left of the limit line in ϵ-V space will map to

‡K is given in Ref. 4 as 0.000175/mph.

points above this limit. However, it is more instructive to plot V/ϵ vs V. This maps the limit line into an upper bound,§ which seems more in consonance with the notion of a limit. Then $(V/\epsilon)_{max}$ has the dimensions of velocity, having a numerical value computed to be $1/K=(V/\epsilon)_{max}=5714$ mph = 8380 ft/s = 4962 knots = 2554 m/s.

Now, the performance of any vehicle lying to the left of the limit line in ϵ-V space will map to a curve lying below the $(V/\epsilon)_{max}$ line in V/ϵ-V space, and so there is a maximum characteristic velocity for any vehicle (not to be confused with its actual velocity) given by $V/\epsilon = WV^2/P = (VL/D)_{max}$. From previous results, we get for aircraft,

$$(VL/D)^* = (V/\epsilon)^* = (3^{3/4}/4)\,(1/A^3B)^{1/4}$$

which, when divided by $(V/\epsilon)_{max}$, yields a number ranging from zero to unity that is now designated as the cruise efficiency C, given by

$$C = 0.57K(A^3B)^{-1/4} \tag{6}$$

This is an absolute measure of aircraft cruise efficiency, since it compares the maximum product of velocity and L/D ratio of the aircraft to the maximum imposed by technology.

To complete this discussion, it is noted that the unit power required at optimum cruise can also be written as a function of the A and B parameters as follows:

$$P_s^* = (DV/W)^* = AV^{*3} + B/V^* = 4(AB^3/3)^{1/4} \tag{7}$$

from Eq. (1), since $V^* = (3B/A)^{1/4}$ from above.

Application of Results

To illustrate the application of the preceding analyses, a parametric survey of some 111 production aircraft was made, representing the bulk of all "cruising" aircraft certificated in the United States which are powered by reciprocating engines, having gross weights less than about 8000 lb (35.6 kN). Training aircraft were included in this survey, but agricultural were not, owing to their highly specialized nature.

This survey was undertaken specifically in order to answer two questions. The first has to do with the current state of design technology: how efficient are these aircraft, in the sense previously defined? The second addressed the question of operational practice: how closely do manufacturers' power recommendations match the cruise optimum?

Data for this survey were drawn from two principal sources, listed in Refs. 6 and 7. The equivalent parasite area required to calculate the A values were obtained by making assumptions for propeller efficiency (uniformly adopted to be 85%) and efficiency factor e of 0.78. These values were adopted as the most probable, based on published work.[8,9] In so doing, some aircraft were undoubtedly favored, and others penalized; however, since the induced drag is only about 25% of the total drag for most aircraft at cruise, the resulting values of f thus obtained were not particularly sensitive to e, as study showed. It is noted in passing that, since data were available for all aircraft at both the 65% and 75% cruise power settings, f could be determined twice for each aircraft. Generally speaking, these two independent determinations yielded remarkable consistency, differing in almost every case by less than 5%. These assumptions become even less influential in the actual determination of cruise performance, since e and f appear in the above relations raised to fractional powers.

§The author is indebted to Dr. D. Jewell for pointing out that this transformation was anticipated by P. Crewe in 1958.[5]

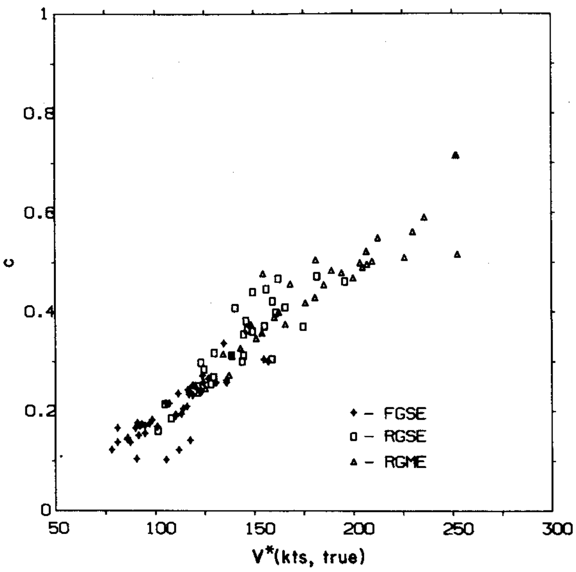

Fig. 3 Cruise efficiency C [Eq. (6)] vs cruise optimum (true) airspeed, 111 piston aircraft. Legend (Figs. 3-5): FGSE—fixed gear, single engine; RGSE—retractable gear, single engine; RGME—retractable gear, multiengine.

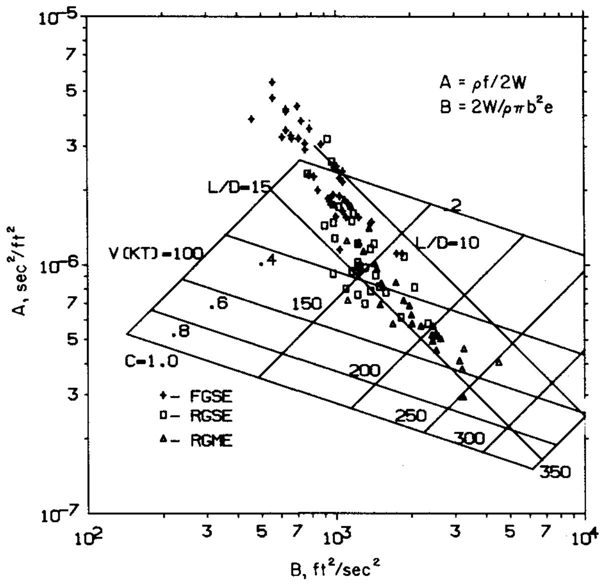

Fig. 4 Plot of parameters A, B, 111 piston aircraft. Overlay grid is cruise optimum airspeed vs cruise efficiency C. Also shown are two lines of $(L/D)_{max}$.

It is also pointed out that the resulting values of A, B, C, and V^* calculated for each aircraft were altitude dependent, that is, they were determined strictly from manufacturers' data, which may be presumed to represent the ideal for each make and model. Some thought was given initially to normalizing all data to a reference altitude, but this was decided against for the reason already stated, and the fact that this would obscure the large gains made in cruise efficiency in recent years with the introduction of turbosupercharged engines in such aircraft.

Figure 3 shows that cruise efficiency varies almost linearly with airspeed, ranging from about 0.15 for simple, fixed-gear training aircraft, to a maximum of 0.7 for high-altitude twins. The explanation for this is that L/D ratios for all aircraft studied fall within the range 9-16 (isolated exceptions noted) and thus cruise efficiency tends to scale directly with velocity. This is brought out clearly in Fig. 4, which is a plot of A vs B.

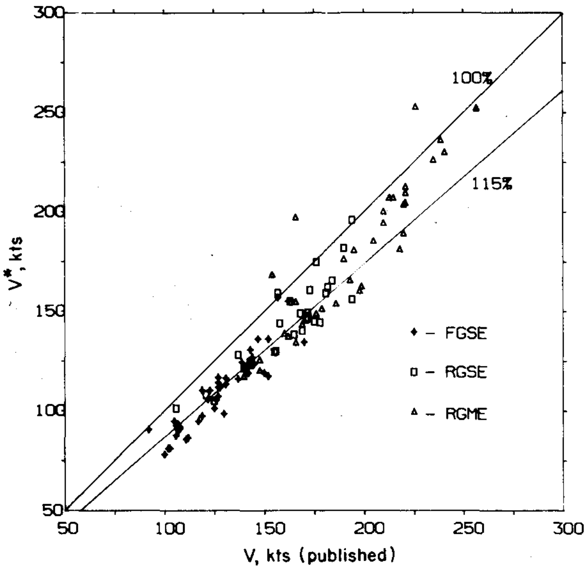

Fig. 5 Comparison of cruise optimum airspeed vs published airspeeds at 75% power, 111 piston aircraft. Percent lines indicate level of correlation.

Shown in this coordinate system are lines of constant V^* and C, with two lines of constant $(L/D)_{max}$ also shown for reference. Among other things, this also illustrates the gains made by supercharging, which permits high-altitude flight. The result is a simultaneous decrease in A and an increase in B, leading to higher speed, and hence cruise efficiency.

The last figure (Fig. 5) shows that published airspeeds are, on the average, about 15% higher than the cruise optimum developed in this study, at the 75% power level. A number of aircraft appear to have engines sized to produce the cruise optimum airspeed at this power level, however, indicating that some existing designs have been nearly optimized in this regard. On the other hand, numerous other aircraft appear to be over-powered at cruise, and would benefit considerably in fuel efficiency if operated at somewhat lower cruise power settings. A plot of published airspeeds against the cruise optimum at the 65% power level leads to essentially the same conclusions, and therefore is not included.

Design for Cruise Efficiency

Recalling that cruise efficiency is a normalized maximum product of L/D ratio and velocity, we return to Fig. 4. Here, it can be seen that to achieve higher efficiencies, there are two obvious paths to follow. The first is a reduction in A at the expense of B. This corresponds to an increase in altitude or gross weight, or a decrease in both f and b^2, all other things constant. This is tantamount to holding L/D constant, and evidently is the tack that has been taken in past design practice. The second path is that of holding V constant, and increasing L/D. This requires (again, all other things constant) a reduction in f, and an increase in b^2. Of the two paths, the latter is obviously the more difficult to achieve. None of this is, of course, very surprising to designers, who have known for a long time that performance is acutely dependent upon parasite area and aspect ratio, reflected through b^2. However, the lucidity with which these facts are illustrated by the present analysis might be considered one of its chief virtues.

But quite apart from this, Fig. 4 can be used as a direct aid to design. Aircraft design is traditionally an iterative process; that is, given a cruise specification, the problem is to determine the physical and aerodynamic characteristics of the aircraft, and the powerplant needed to achieve this specification. However, if the specification were given in terms of cruise efficiency, optimum cruise velocity, and the power available at cruise, then, with the aid of this figure, the physical and aerodynamic characteristics can be readily determined. As an example, suppose a light twin is to be designed to have a cruise efficiency of 0.6 at 25,000 ft (7600 m) and an optimum cruise speed of 250 knots. The aircraft is specified a priori to be powered by two engines rated at 300 hp (225 kW) each, and it is intended that optimum cruise take place at 65% power. As a first estimate, the propeller efficiency may be taken as 85%, and the efficiency factor as 0.7. Then, from Fig. 4, we read $A = 3.5\times10^{-7}$ s^2/ft^2, $B = 4\times10^3$ ft^2/s^2. With these inputs and the help of Eqs. (7), (2), and (3), we immediately compute $W = 4900$ lb (21.9 kN), $f = 3.43$ ft^2 (0.319 m^2), and $b = 32.25$ ft (9.83 m), which should be well within present design capabilities; the point is that the designer now has the necessary parameters which, if reached, guarantee that the cruise specification will be met. Note that this is not an iterative process, but rather a straightforward approach, greatly reducing the need for expensive and time-consuming tradeoff studies.

Summary and Conclusions

It is axiomatic that the basis of any physical theory is a set of experimental results. From this it follows that when theory and observation conflict, the theory must be reexamined. In this paper we have all but abandoned the classical formulation of aircraft cruise performance since, from a practical standpoint, it has little more than academic interest. The optimum fuel efficient airspeed predicted by classical theory is completely at odds with design and operational philosophies.

In an attempt to understand why such a disparity exists, it was found that while the definition of the "best" aircraft is arguable, there is, unarguably, a best *method* of operating a given aircraft which results in a maximum rate of return in airspeed (hence reduction in flying time) rather than in distance traveled, per unit of fuel consumed. von Kármán must surely have had this in mind when he wrote, in later explanation of his work with Gabrielli[10]: "If the specific power is proportional to speed, the total work necessary for transportation over a given distance is the same. This condition corresponds to straight lines of 45-deg slope in the logarithmic diagram. We can therefore say that any vehicle performs best where its curve has a 45-deg slope. If the slope is less than 45 deg, the vehicle is improved by increasing speed. If the slope is greater than 45 deg, this is a sign that the vehicle is beyond its best application." From this, it is evident that any vehicle, aircraft included, will be operated at its "best application" when the product of speed and L/D ratio (or WV^2/P) is maximum. For propeller aircraft, this corresponds to the optimum utilization of excess power for cruising purposes, as has been demonstrated.

It is the author's belief that this work has immediate application. As a start, manufacturers (who know the physical and aerodynamic parameters of their products better than anyone else) might consider supplementing their operational data with the information needed by pilots to operate at the cruise optimum developed in this paper.

In conclusion, only cruise performance has been addressed in this paper; there are obviously other performance requirements that any design must satisfy. However, an aircraft designed for good cruise efficiency will probably excel in these other areas as well. It is hoped that the rationale presented herein will result in better fuel utilization of existing aircraft, and will point the way in which future design emphasis ought to proceed.

Acknowledgments

It is a pleasure to acknowledge the great assistance rendered during the course of this study by Ensign Jon A. Buttram, USN, who was responsible for the data accumulation and reduction presented in this paper. The author also acknowledges the assistance of the U.S. Naval Academy

Computer-Aided Design group in the preparation of the graphics, and also his colleagues and fellow pilots for their encouragement to carry this work to completion. Mrs. Morva Hamalainen deserves special thanks for typing the manuscript.

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